aeroelastic effects or thrust-axis offset can have a larger influence on the characteristics of supersonic long-period motions than thrust/speed effects alone, including large changes in ω_{n_p} and τ_z, as well as negative (unstable) values of ζ_p. This moment-producing characteristic provides additional reason to consider the speed-to-moment/altitude-to-thrust controller. Short-period stability augmentation may have a beneficial effect on long-period stability, although overall stability can be assured only by coordinated design of the full-order control system.

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Reply by Author to Robert F. Stengel

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THE preceding Technical Comment of Stengel[1] provides valuable additional information about thrust/speed and pitching moment/speed effects and helps to clarify supersonic flight dynamics characteristics that show significant differences when compared with well-known relationships in subsonic flight. More information and further insight into the problems of supersonic flight are provided by a comprehensive paper of the same author in Ref. 2 that includes the effects of control-loop closures and related favorable control laws and the effects of atmospheric disturbances. In recent papers,[3-5] additional topics concerning the effect of thrust characteristics and their significance for supersonic and hypersonic flight are considered.

The central issue of Ref. 6 concerns the direct thrust/speed effects on long-period dynamics in supersonic flight and their differences as regards well-known characteristics attributed to these effects. Particular emphasis is given to the phugoid and its insensitivity concerning the effect of thrust changes with speed. This effect can be qualified as negligible in supersonic flight or (practically) not existent. It pronouncedly contrasts with the well-known subsonic characteristics according to which the phugoid is substantially influenced by the thrust variations with speed. It is the purpose of this reply to provide a more detailed insight which helps to point out more clearly the central issue of Ref. 6.

In Ref. 6, the root locus technique was applied for showing the effect of thrust changes with speed. From the results presented, it follows that the maximum change in phugoid damping ratio for the whole root locus is on the order of $\zeta_{p\max} - \zeta_{p\min} = 0.02$. This holds for an unlimited gain of the root locus, i.e., for hypothetical thrust changes of unlimited size. Compared to such large thrust changes, a maximum change in damping ratio $\zeta_{p\max} - \zeta_{p\min} = 0.02$ may be qualified as negligible. This contrasts with the substantial effect of thrust/speed dependence in subsonic flight where the change in phugoid damping ratio reaches the maximum value possible $\zeta_{p\max} = 1$ (i.e., the phugoid is changed from an oscillation lightly damped to an aperiodic mode of motion). This already holds for moderate gains (see Fig. 1 in Ref. 6).

A simplified analysis may provide more insight into the problem addressed by considering analytical expressions for the long-period modes of motion. An adequate mathematical model for a horizontal reference flight condition in the absence of significant pitching moments may be expressed as (with the notation used in Ref. 1)

$$\begin{bmatrix} \Delta\dot{V} \\ \Delta\dot{\gamma} \\ \Delta\dot{z} \end{bmatrix} = \begin{bmatrix} TD_V & -g & TD_z \\ L_V/V_n & 0 & L_z/V_n \\ 0 & -V_n & 0 \end{bmatrix} \begin{bmatrix} \Delta V \\ \Delta\gamma \\ \Delta z \end{bmatrix} + \begin{bmatrix} T_{\delta T} \\ 0 \\ 0 \end{bmatrix} \delta T \quad (1)$$

The characteristic equation related to this system is

$$\Delta(s) = s^3 + Bs^2 + Cs + D = 0 \quad (2a)$$

where

$$B = -TD_V, \quad C = \frac{gL_V}{V_n} + L_z, \quad D = L_V TD_z - L_z TD_V \quad (2b)$$

There are three roots $s_{1,2,3}$ of which one usually is real valued ($s_1 := s_h$: height mode) and the others are complex ($s_{2,3} := -\zeta_p \omega_{n_p} \pm i\omega_{n_p}\sqrt{1-\zeta_p^2}$: phugoid). As regards their relative magnitude, the following relations usually hold:

$$|s_h| \ll |\omega_{n_p}|, \qquad |\zeta_p| \ll 1 \quad (3)$$

The connection between the roots and the coefficients of the characteristic equation is given by

$$s_h - 2\zeta_p\omega_{n_p} = -B, \quad \omega_{n_p}^2 - 2s_h\zeta_p\omega_{n_p} = C, \quad s_h\omega_{n_p}^2 = -D \quad (4)$$

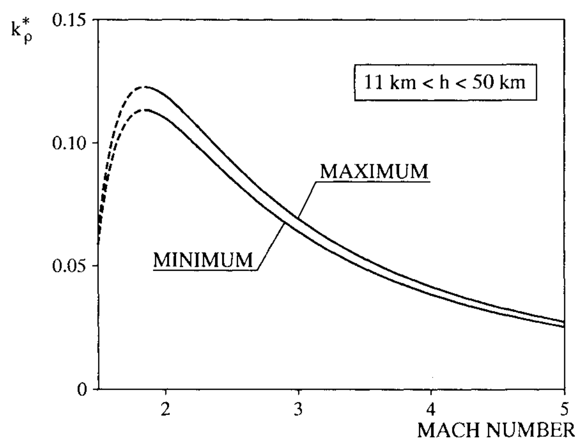

Fig. 1 Factor k_p^* as a function of Mach number (atmospheric data from Ref. 7).

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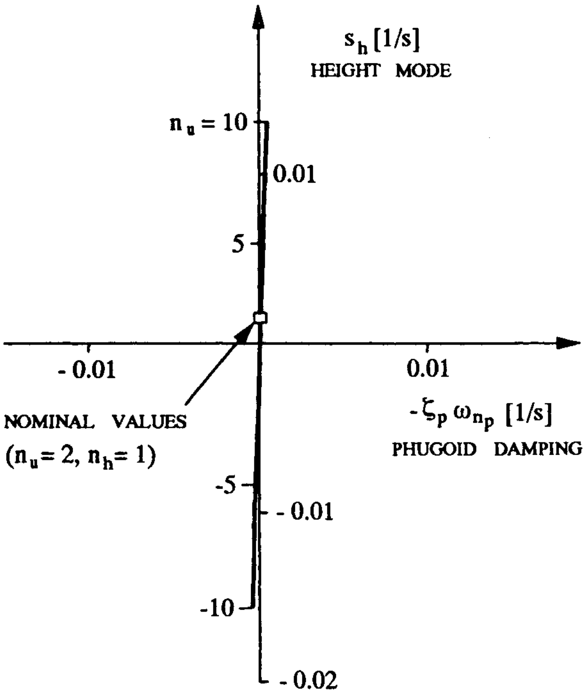
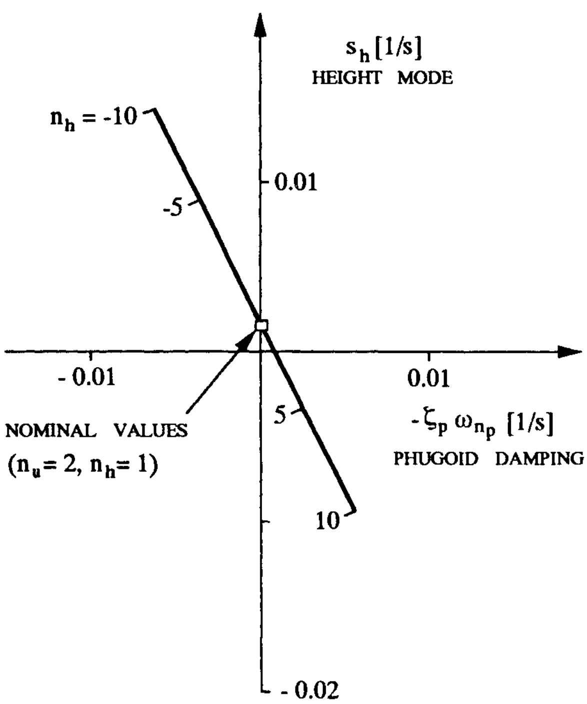


a) Thrust/speed dependence



b) Thrust/altitude dependence

Fig. 2 Effect of thrust/speed and thrust/altitude dependence on phugoid damping and height mode in supersonic flight ($M_0 = 3$). $n_u = mV_nTD_V/D_n$ and $n_h = m(\rho_n/\rho_z)TD_z/D_n$.

Applying the magnitude relation expressed by Eq. (3), the following approximations can be derived:

$$\omega_{n_p} \approx \sqrt{C} = \sqrt{(gL_V/V_n) + L_z}$$

$$s_h = -D/\omega_{n_p}^2 \approx -D/C$$

$$= -(L_VTD_z - L_zTD_V)/(gL_V/V_n + L_z) \qquad (5)$$

$$\zeta_p = \frac{1}{2\omega_{n_p}}(B + s_h) \approx \frac{1}{2\sqrt{C}}\left(B - \frac{D}{C}\right)$$

$$= \frac{1}{2\omega_{n_p}}\left[-TD_V - \frac{L_VTD_z - L_zTD_V}{gL_V/V_n + L_z}\right]$$

These equations can be expanded further when considering that the derivative combination $gL_V/V_n + L_z$ can be simplified because it is dominated by L_z in supersonic flight. With the use of

$$L_V = [2/V_n + C_{L_V}/C_{L_n}]g, \qquad L_z = g(\rho_z/\rho_n)$$

the derivative combination addressed may be written as

$$g\frac{L_V}{V_n} + L_z = L_z\left[1 + \frac{g}{V_n^2}\frac{\rho_n}{\rho_z}\left(2 + V_n\frac{C_{L_V}}{C_{L_n}}\right)\right] \qquad (6)$$

The second part of the bracketed term

$$k_\rho^* = \frac{g}{V_n^2}\frac{\rho_n}{\rho_z}\left(2 + V_n\frac{C_{L_V}}{C_{L_n}}\right) \qquad (7)$$

can be considered as small in supersonic flight, i.e.,

$$k_\rho^* \ll 1 \qquad (8)$$

This is illustrated in Fig. 1 (with the Ackeret rule applied for C_{L_V}). From Fig. 1, it follows that the relation described in Eq. (8) holds for the whole Mach number and altitude range of interest.

Applying $k_\rho^* \ll 1$ yields for the derivative combination addressed

$$\frac{1}{gL_V/L_n + L_z} \approx \frac{1}{L_z}(1 - k_\rho^*), \quad \sqrt{\frac{gL_V}{L_n} + L_z} \approx \left(1 + \frac{k_\rho^*}{2}\right)\sqrt{L_z}$$

With the use of these relations, the expressions of Eq. (5) may be rewritten as

$$\omega_{n_p} \approx (1 + k_\rho^*/2)\sqrt{L_z} \qquad (9a)$$

$$\zeta_p \approx -1/(2\omega_{n_p})\left[k_\rho^*TD_V + (1 - k_\rho^*)(L_V/L_z)TD_z\right] \qquad (9b)$$

$$s_h \approx (1 - k_\rho^*)\left[TD_V - (L_V/L_z)TD_z\right] \qquad (9c)$$

Equation (9b) shows that the effect of thrust/speed dependence TD_V on phugoid damping is substantially reduced because it is multiplied with the small factor $k_\rho^* \ll 1$. This is illustrated in Fig. 2, which shows that there is nearly no change in phugoid damping (with n_u used for describing the effect of thrust/speed dependence TD_V).

For moderate TD_V values as used in Fig. 2, the approximation (9b) agrees well with the results of the higher-order model fully accounting for longitudinal dynamics. For larger values of TD_V, the effect of thrust/speed dependence on phugoid dynamics is further reduced and is practically zero as shown in Ref. 6 (Fig. 3) with the use of the root locus technique.

From Eq. (9c), it follows that the height mode is very sensitive to thrust/speed dependence because the effect of TD_V is not reduced here. This is also illustrated in Fig. 2, which makes evident the contrast in the sensitivities of the two modes of motion as regards the effect of TD_V.

It may be of interest to compare the results concerning TD_V with the effect of thrust/altitude dependence TD_z, which is the other direct thrust influence. This is also illustrated in Fig. 2 from which it follows that the height mode and phugoid now show a similar degree of sensitivity (with n_h used for describing the effect of thrust/altitude dependence TD_z). As a consequence, a stabilization of the height mode is combined with a comparable destabilization of the phugoid and vice versa.

The above considerations are concerned with direct thrust effects that deal with force characteristics only. There may also be an influence on pitching moment due to thrust/axis offset. This can have an influence that may be more significant than the effects considered above (Refs. 2 and 8).

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